Using convolutions to calculate Γ_1

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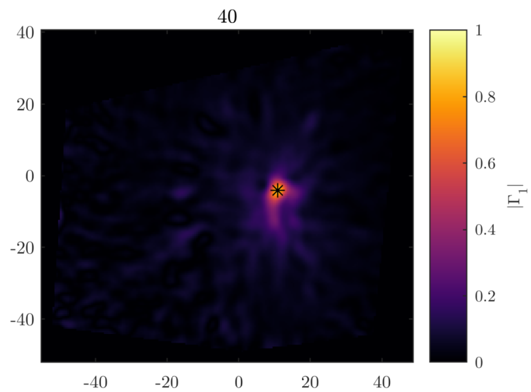

Figure 1: Γ_1 field of a vortex obtained through high-speed SPIV and its tracked location (black star) at peak Γ_1.

As defined by Graftieaux (2001):

$$\Gamma_1(P) = \frac{1}{S}\int_{M\in S} \frac{(\vec{PM}\times\vec{U_M})\cdot\vec{z}}{||\vec{PM}||\cdot||\vec{U_M}||}dS = \frac{1}{S}\int_S \sin(\theta_M)dS \tag{1}$$

In a discrete domain, the expression can be written as an average:

$$\Gamma_1(P) = \frac{1}{N}\sum_{M\in S} \frac{(\vec{PM}\times\vec{U_M})\cdot\vec{z}}{||\vec{PM}||\cdot||\vec{U_M}||} \tag{2}$$

Narrowing down to a 2D case, the cross product dotted with $\vec{z} = [0\ 0\ 1]$ will be:

$$\Gamma_1(P) = \frac{1}{N}\sum_{M\in S} \frac{(PM_x U_{M,y} - PM_y U_{M,x})}{||\vec{PM}||\cdot||\vec{U_M}||} \tag{3}$$

The expression can then be separated into two convolutions:

$$\Gamma_1(P) = \frac{1}{N}\left[\sum \frac{PM_x U_{M,y}}{||\vec{PM}||\cdot||\vec{U_M}||} - \sum \frac{PM_y U_{M,x}}{||\vec{PM}||\cdot||\vec{U_M}||}\right] \tag{4}$$

$$\Gamma_1(P) = \frac{1}{N}\left[\sum \frac{PM_x}{||\vec{PM}||}\frac{U_{M,y}}{||\vec{U_M}||} - \sum \frac{PM_y}{||\vec{PM}||}\frac{U_{M,x}}{||\vec{U_M}||}\right] \tag{5}$$

Where the fractions are to be convolved. The convolution kernel can be as large as needed and can be generated prior to the calculation. Also, because the kernel is the x and y component of a unit vector, a simple grid of integers can be used for the discrete case, regardless of the actual grid spacing.

The improvements applied to this code reduced the computational time in Matlab by about 3 orders of magnitude.